EXTENDED MODELLING AND NUMERICAL INVESTIGATION OF PHASED ARRAY SYNTHETIC JET CROSS-FLOW INTERACTIONS

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Abstract

A classical analysis of incompressible unsteady Navier-Stokes equation have been under discussion for a long time by various reasearchers, it has been found that the exact analytical unique solution existed only below and undefined Reynolds number limit. Several solutions were existed for a range of Reynolds number above the limit, and that has no solution exsisted above the second undefined Reynolds number (i.e, solution enter into turbulent region and one has to solve another set of equations for turbulance). A useful study was done for two dimensional unsteady incompressible flow in which vorticity is propotional to stream function perturbed by uniform stream. Recently this unsteady sloution was used to analyse the flow behaviour on flat plate using Synthetic Jet Actuators (SJA) which consists of oscillating membrane expels in fluid through orifice. The results obtained at different time level were presented and it was found as the time progresses streamlines, on the flat plate with number of actuators placed in an array, become flatter caused rapidly decay in vortices on trailing edge. The authentication of these results were needded to be validated through experiment or numerical simulation. In this paper, an attempt has been made to solve incompressible unsteady Navier-Stokes equations numerically to analyse the behaviour of single and multiple sythetic jet actuator in a cross flow conditions. The behaviour of SJA implemented by imposing a special kind of boundary condition on the bottom of flate plate was studied. Results have been obtained for various Reynolds numbers and were presented. These numerical results were in close agreement with [1].

Keywords: Incompressible unsteady Navier-Stokes equations, two dimensional flat Plate, Cell centered discrete Navier-Stokes equations, SOR solution method

Introduction

In [1], an attempt was made to predict the flow condition on an aero-plane wing by introducing SJA through the exact analytical solution of Navier-Stokes equations. The exact analytical solutions of linearized Navier-Stokes equations have been obtained and observed that the nonlinear convective term in the Navier-Stokes equations vanished when vorticity is the function of stream function alone or proportional to stream function [2]. In [2], it has also been observed an exact analytical solution that represent a double infinite array vortices decaying exponentially with time. The same was used with minor modification for the reverse flow about the flat plate with suction[4] .

In [1], we have attempted to predict the influencing fluid flow through suction and blowing with the help of exact analytical solutions of Navier-Stokes equations. The suction and blowing were created by SJA, a device that is used for zero mass-flux momentum addition to fluid flow. The actuators play an important role as control authority. In present study, it has investigated this control authority through plume core identification and array momentum coefficient.

The study was motivated by application of virtual shaping of flow for example air-foil with shape of streamlines over an aero-foil through a single synthetic jet which produces the lift. In [1], this situation was studied with the help of solution of exact analytical NSEs with special boundary condition having source and sink. Validation of the theoretical results obtained in [1] would be done either by experiment or numerical simulation. Experiment was beyond our scope, therefore, we carried out the validation through numerical experiment.

In present paper, incompressible unsteady Navier-Stokes equations were solved numerically in a two dimensional horizontal channel. A special type of boundary condition in space and time are specified on bottom boundary of the channel which simulates the behavior of actuators. A Conventional SOR (successive over relaxation) method was employed to solve pressure Poisson equation whereas the time dependent momentum equations were discretized on cell centered MAC type grid. The pressures were defined on the cell center and velocities were at cell boundaries. In the following section, we described the details of discretization of NSEs in primitive variables together with discretized equations on a MAC type grid.

Differential and Difference Navier-Stokes Equations

The incompressible unsteady Naiver-Stokes equations in two dimension are given below :

Continuity

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \quad (1)$$

U momentum

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \quad (2)$$

V momentum

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \quad (3)$$

where u, v, are the veloctiy components in x and y directions respectively. P, ρ and μ were pressure, density and viscosity respectively. A MAC staggered grid system is used to locate flow variables.

Discretize Navier-Stokes Equation

The discrete equations were derived by first integrating the differential equations over each control volume surrounding the location of variables. They were expressed for a unit volume and fluxes over each control volume surface were taken to be constant. The discretized Navier-Stokes equation were given as follows;

Continuity equation:

$$u_{i+1/2,j}^{n+1}-u_{i-1/2,j}^{n+1}+v_{i,j+1/2}^{n+1}-v_{i,j-1/2}^{n+1}=0 \quad (4)$$

U momentum:

$$\frac{u_{i+1/2,j}^{n+1}-u_{i-1/2,j}^{n+1}}{\Delta t}=-\left(\frac{1}{h}\left(u^2\right)_{i+1,j}^{n}-\left(u^2\right)_{i,j}^{n}+(uv)_{i+1/2,j+1/2}^{n}-(uv)_{i+1/2,j-1/2}^{n}\right)+ \quad (5)$$

$$\frac{v}{h^2}(u_{i+3/2,j}^{n}+u_{i-1/2,j}^{n}+u_{i+1/2,j+1}^{n}+u_{i+1/2,j-1}^{n}-4u_{i+1/2,j}^{n})-\frac{1}{h}(P_{i+1,j}-P_{i,j})$$

V momentum:

$$\frac{v_{i,j+1/2}^{n+1}-v_{i,j+1/2}^{n}}{\Delta t}=-\left(\frac{1}{h}(uv)_{i+1/2,j+1/2}^{n}\left(u^2\right)_{i+1,j}^{n}-(uv)_{i-1/2,j+1/2}^{n}+\left(v^2\right)_{i,j+1}^{n}-\left(v^2\right)_{i,j}^{n}\right)+\frac{v}{h^2}(v_{i+1,j+1/2}^{n}$$

$$+v_{i-1,j+1/2}^{n}+v_{i,j+3/2}^{n}+v_{i+1/2,j-1/2}^{n}-4v_{i,j+1/2}^{n})-\frac{1}{h}(P_{i,j+1}-P_{i,j})$$

(6)

Solution Algorithm

The resulting finite difference set of algebraic equations (4 - 6) were solved as follows

i. ‘u’ and ‘v’ momentum equations were first solved on an intermediate time level using the advection and diffusion terms only.

ii. Find the pressure needed to make velocity field incompressible.

iii. Correct the velocity by adding the pressure gradient.

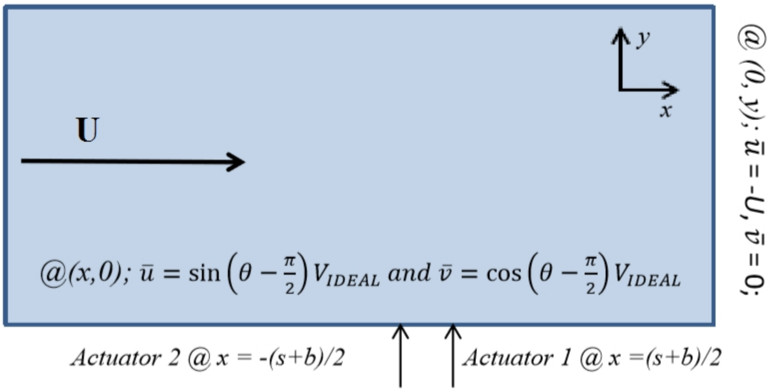
iv. Solve Poisson equation for pressure using SOR method.

v. The algorithm continued for a longer time till the solution were converged.

Boundary Condition

Consider a two dimensional flat plate problem with cross-flow, the boundary condition were specified as below:

Fig 1. Boundary Conditions with two actuators



$$V_{IDEAL} = V_0 \sin(kt^* + \varphi)\, e^{-\frac{\left(x-\frac{s+b}{2}\right)^2}{2p^2}} + V_0 \sin(kt^*)\, e^{-\frac{\left(x+\left(\frac{s+b}{2}\right)\right)^2}{2p^2}}$$

Results and Discussion

Equations (4 - 6) were solved for various Reynolds numbers on two dimensional flat plate with actuators as a suction and ejection placed at the bottom of the plate. The results were shown in figures ().

In figure (2) streamlines and velocity vector were plotted for Reynolds number 100. It was shown that the streamlines were converged at the actuators and were remained live even for a longer time with a total number of iterations 10,000.

In contest, figure (3), was shown that for Reynolds number 1000 the vortices were vanished gradually due to the effect of actuators at time 0.3 seconds. The same behavior was found in figure (4) for Reynolds number 10,000 and the vortices were vanished more rapidly at time of 0.2 seconds.

Figures (5 - 6) were a vector plot for Reynolds number 1000 and 10,000 also shown the recirculation of flow near the actuators.

Figure (7) was a plot of time of vanishing vortices vs Reynolds numbers. It was shown that the vortices at high Reynolds numbers vanished more quickly. It was concluded that the vortices created close to the actuators were died down before reaching to the trailing edge which controlled the cross flow condition. It was suggested that if an array of actuators were placed in proper positions on a flat wing would control cross flow condition quite efficiently. The same observation would be validated by performing an experiment by placing actuators on an airplane wing. It was further concluded that the result would be better if we considered a three dimensional case in which the bottom surface would be considered for actuators in an array.

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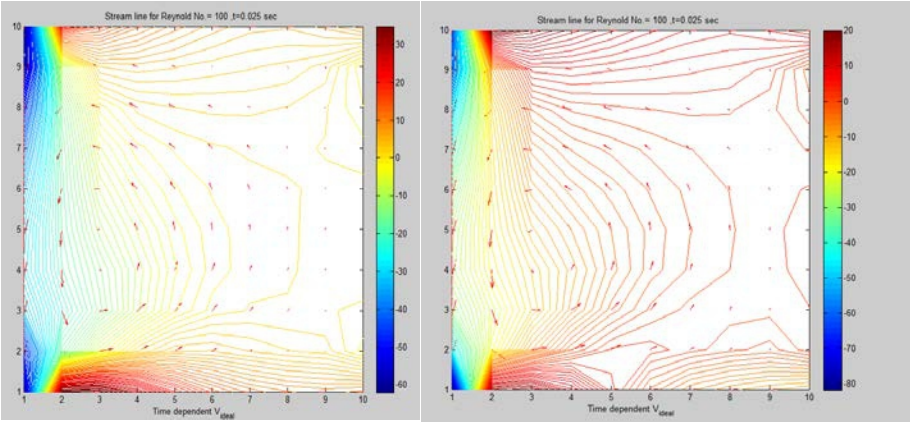

Fig 2 Streamlines for Reynold no.100 at t=2.5 sec and t=250 sec

Reynolds No	Time of vanishing Vortices (sec)
170	2
200	1.075
250	0.62
300	0.55
400	0.45
1000	0.3
10000	0.2

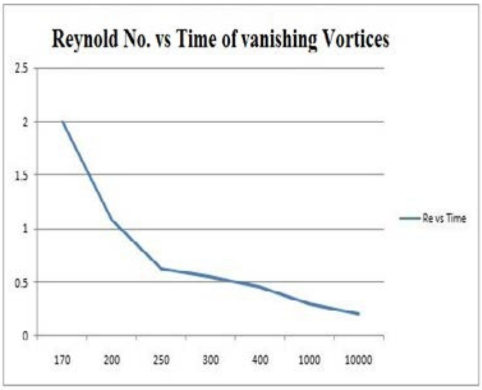

Fig 3 Table and Graph of Reynolds No. vs Time Vanishing Vortices

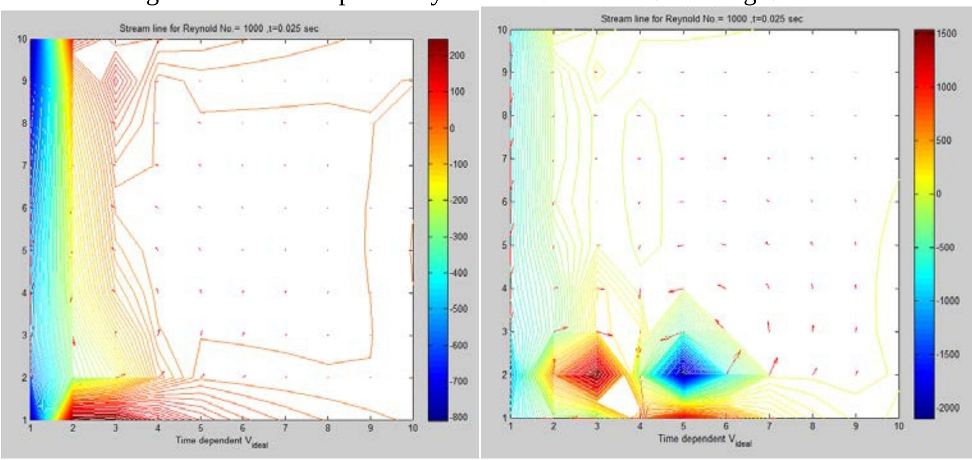

Fig 4 Streamlines for Reynold no. 1000 at t=0.125 sec. and t=0.25 sec.

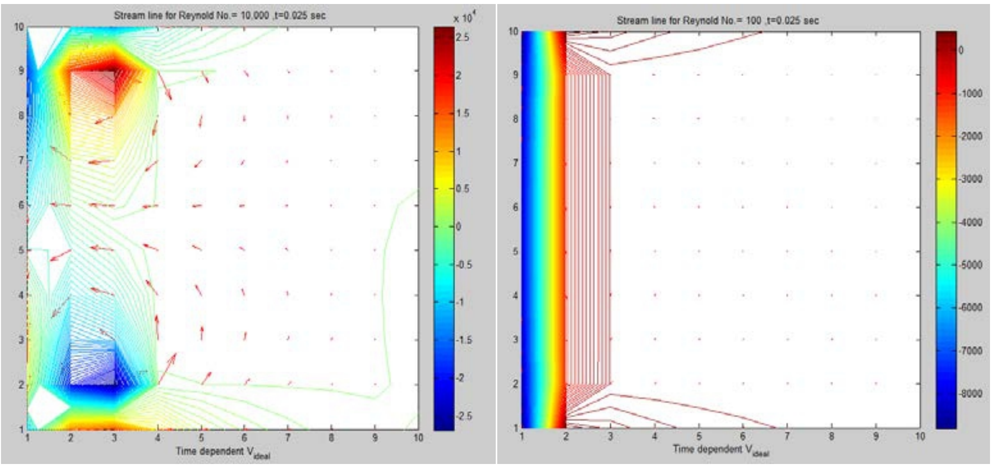

Fig 5 Streamlines for Reynold no. 10,000 at t=0.075 sec and t=0.125 sec.

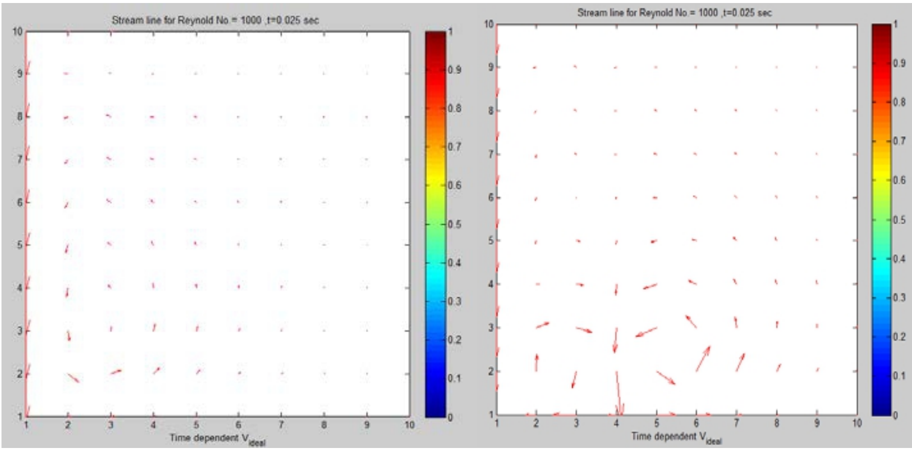

Fig 6 Vector plot for reynold no 1000 at t=0.125 sec and t=0.25 sec

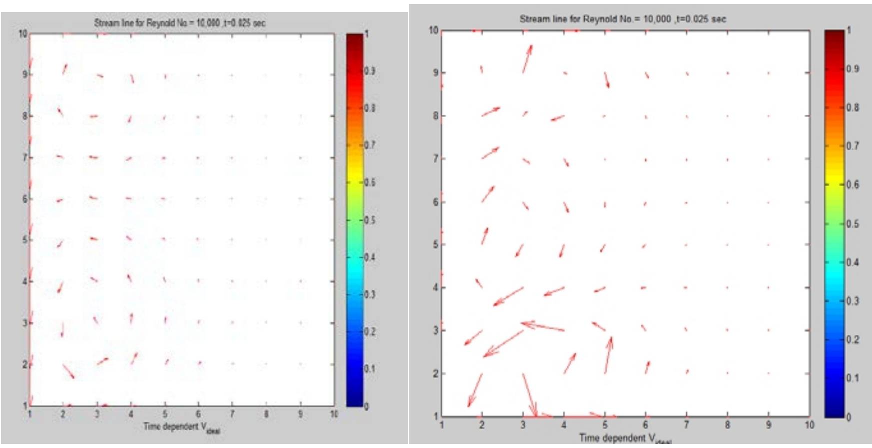

Fig 7 Vector plot for reynold no 1000 at t=0.075sec and t=0.25sec.