REVIEW OF MODELING PREFERENCES FOR DECISION MODELS

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Abstract

A group decision problem is set in environments where there is a common issue to solve, a set of possible options to choose, and a set of individuals who are experts and express their opinions about the set of possible alternatives with the intention to reach a collective decision as the unique solution of the problem in question. The modeling of the preferences of the decision-maker is an essential stage in the construction of models used in the theory of decision, operations research, economics, etc. On decision problems experts use models of representation of preferences that are close to their disciplines or fields of work. The structures of information most commonly used for the representation of the preferences of experts are vectors of utility, orders of preference and preference relations. In decision problems, the expression of preferences domain is the domain of information used by the experts to express their preferences, the main are numerical, linguistic, and intervalar stressing the multi-granular linguistic. This paper is a review of these concepts. Its purpose is to provide a guide of bibliographic references for these concepts, which are briefly discussed in this document.

Keywords: Preference modeling; decision models; utility vector; orders of preference; preference relations

Introduction

A group decision problem sets in environments where there is a common issue to solve, a set of possible options to choose, and a set of individuals who are experts that express their views on the set of possible alternatives, and intend to reach a collective decision as the unique solution of the problem in question (Van De Ven & Delbecq, 1974), (Kacprzyk, 1986), (Peláez Sánchez, 2000).

Decision problems are divided into two large groups: those based on preferences and the other based on similarity (Perny, 1998).

Decision problems based on *preferences* include those who belong to the denomination of Roy (Roy and Vanderpooten, 1995), (Roy, 1996): 1) $P\alpha$, given the set of alternatives *X*, get the smaller subset of alternatives *X'* (*X'* $\subset$ *X*) so that it can be justified to ignore any *x* ϵ *X*-*X'*; 2) $P\gamma$, given the set of alternatives *X*, sort the array *X* in equivalence classes in decreasing sense of quality.

Similarity-based decision problems are divided into: 1) given the set *X* of alternatives, associate each object *X*, with a set of default, absolute categories in the sense that does not depend on *X* (classification); y 2) given the set of alternatives *X*, assign *X* objects into groupings that do not exist in advance, so that they can justify the similarity of an object that are grouped with it, and the difference with the other (clustering).

In the classical theory of decision (Keeney & Raiffa, 1976), (Howard & Matheson, 1984), the European School of MCDA (Multicriteria Decision Aid) (Roy, 1990), (Roy, 1996), and methods that are based on a paradigm of learning by examples (Greco, Matarazzo & Slowinski, 2001), (Greco, Matarazzo & Slowinski, 2002), (Fernandez, Navarro & Duarte, 2007), emphasizes the central role of the modeling of the subjectivity of the DM (decision maker). It decides the conflict of attributes, in the assessment of risks, and in interpersonal conflict situations, taking into account that a decision problem is objective by the set *X* of alternatives and the consequences of its elements; but it is subjective for the evaluation of the consequences and their reflection in the mind of the DM (Fernández & Olmedo, 2007).

This modelling of the preferences of the decision-maker is an essential stage in the construction of models used in decision theory, operations research, economics, etc. (Fernández Barberis, Escribano Ródenas & Calvo Martín, 1997).

One aspect to take into account when modeling the preferences in social choice problems is the problem of *rationality* (Arrow, 1951). It considered a collective of individuals who must decide among several options, taking into account individual preferences and the aggregation of these preferences has to follow certain rules of consistency or rationality. This formalized the notion of preference between pairs of objects through the *binary relationship* concept, incorporating the notion of "rational behavior" by requiring that each individual preference constitutes a total preorder (this means that the relationship of preference must be reflexive, transitive and complete (or total)).

Another alternative approach is to base the notion of rationality on *functions of choice* instead of binary relations (May, 1954). With this idea, the representation of rationality is made through axioms about election functions which does not necessarily come from preference relations.

This article is a brief review about the modelling of preferences in decision support systems that been structured in the following way: the main aspects related to the modelling of preferences will be summarized in section 2. Section 3 will describe the main structures used for the representation of the preferences, section 4 will present the main domains of expression of preferences, the main conclusions will be indicated in section 5, ending with the acknowledgements and references.

Modeling Preferences:

Modeling Preferences is one of the essential activities in decision making problems. Experts on the basis of their knowledge, experiences and beliefs have to issue their valuations on the set of alternatives and establish an order of precedence over the suitability of each of them as a solution to the problem. About the problems of decisions system experts we can say that they use models of representation of preferences which are related to their disciplines or fields of work. For example, experts belonging to technical areas may feel very comfortable representing their preferences by using numerical values. However, experts who belong to other less technical disciplines, such as those belonging to social areas, may prefer to express their preference using expressions closer to human language such as words or linguistic terms. To cope with this type of ratings are defined different mechanisms allowing to transform the preferences of experts in formal representations that support mathematical, rational and consistent treatment of such information (Fortemps & Slowinski, 2002), (Oztürk, Tsoukiàs & Vincke, 2005), (Perny & Tsoukiàs, 1998), (Roubens & Vincke, 1985), (Armstrong, 1948), (Debreu, 1959), (Capurso & Tsoukiàs, 2003), (Coombs & Smith, 1973), (Kahneman, Slovic & Tversky, 1981), (Xu, 2014), (Hu et al., 2014), (Sánchez Sánchez, 2007).

Modeling Preferences is an area of work within decision making dedicated to the representation of the preferences of experts. The way to express preferences is very important in the aggregation operators (Barzilai, 2010), (Doyle, 2004), (Oztürk, Tsoukiàs & Vincke, 2005), (Roubens & Vincke, 1985), (Liu, Zhang & Zhang, 2014), (La Red Martínez & Pinto, 2015). In this respect, two points of view are presented:

- The structure of information used by experts for the representation of preferences.
- The domain of information in which the preferences are expressed on the set of alternatives of the problem.

Structures for the Representation of Preferences

The information structures most commonly used for representing the preferences of experts are the following (Herrera-Viedma, Herrera, & Chiclana, 2002), (Nurmi, 1988), (Tanino, 1990):

- Utility vectors.
- Orders of preference.
- Preference relations.

Utility Vectors

Utility vectors have been a structure of representation of information used in the classical literature to represent the preferences of experts (Dombi, 1995), (Luce & Suppes, 1965), (Martínez, 2007), (Tanino, 1990). It is a very simple structure based on a vector where each element is interpreted as a preference or utility of one of the alternatives of the problem (Sánchez Sánchez, 2007). Set out in the following manner:

Let be $E = \{e_1,\ldots, e_m\}$ ($m\geq 2$) a finite set of experts who have expressed their preferences on a finite set of alternatives $X = \{x_1, x_2,\ldots, x_n\}$ ($n\geq 2$). The preferences given by the experts on the set of alternatives X using utility vectors U^i would be the following: $U^i = \{u^i_1,\ldots,u^i_n\}$, where u^i_j is the utility or assessment given by the expert I to the alternative j. It is assumed that the higher the value of u^i_j, more meets the alternative $j$the objective of the problem in the view of the expert i.

Orders of Preference

This structure establishes a ranking or an order of alternatives that represents the suitability of each as a solution to the problem in accordance with the terms of the point of view of each expert (Nurmi, 1988), (Seo & Sakawa, 1985), (Tanino, 1984).

An order of preference O^i represents an order given by the expert I on the set of alternatives X according to your preferences. It is represented by an decreasing ordered vector of the set of alternatives: $O^i = \{o^i(1),\ldots, o^i(n)\}$. For any order of preference O^i assumes that the lower is the position of an alternative in that order, this alternative is more preferred than the rest to solve the problem in the opinion of the expert i.

Preference Relations

Preferences on a set of alternatives $X = \{x_1,\ldots, x_n\}$ can be modeled as binary alternative-peer relations x_lRx_k ($x_1, x_k \in X$), which are interpreted as the intensity or the degree of preference of the alternative x_l on the alternative x_k (Roubens & Vincke, 1985).

When the sets of alternatives are finite, the preference relations are information infrastructures capable of supporting this type of binary relationships between alternatives. It is possible to use a matrix representation of the preferences of the decision makers (Lee & O'Mahony, 2005), (Tanino, 1990), (Yue, Yao & Zhang, 2005).

Experts express preferences over the set of alternatives *X* using preference relations valued in [0, 1] (Chen &Hwang, 1992), (Fodor & Roubens, 1994), (Kacprzyk, Nurmi & Fedrizzi, 1997), (Xu, 2005b), (Xu, 2006).

In decision problems, it is important that the opinions of the experts are consistent, which requires that preference relations met reciprocity, completeness and transitivity properties (Herrera, Martínez & Sánchez, 2005), (Salles, 1998).

Preference relations have been successfully used by many authors to solve group decision problems (Fan & Chen, 2005), (Herrera, Herrera-Viedma & Verdegay, 1996), (Herrera, Martínez & Sánchez, 2005), (Kacprzyk, 1987), (Kacprzyk, Fedrizzi & Nurmi, 1992), (Xu, 2004a), (Xu, 2005a), (Sánchez Sánchez, 2007).

Domains of Expression of Preferences:

In decision problems, the domain of expression of preferences means the domain of information used by the experts to express their preferences.

The literature shows that, in most decision-making problems, experts express their preferences in the same domain of information, speaking of problems defined in *homogeneous contexts* (Arfi, 2006), (Ben-Arieh & Zhifeng, 2006), (Bordogna, Fedrizzi & Pasi, 1997), (Carlsson & Fuller, 2001), (Delgado, Vendegay & Vila, 1992), (Fan, Ma & Zhang, 2002), (Herrera & Herrera-Viedma, 2000), (Herrera, Herrera-Viedma & Verdegay, 1995), (Lee, 1999), (Li & Yang, 2003), (Marimin, Umano, Hatono & Tamura, 1998), (Rasmy, Lee, Abd El-Wahed, Ragab & El-Sherbiny, 2002), (Xu, 2004b), and some problems in which experts used different information domains, known as problems defined in *heterogeneous contexts* (Delgado, Herrera, Herrera-Viedma & Martínez, 1998), (Fan, Xiao & Hu, 2004), (Herrera & Martínez, 2001a), (Herrera, Martínez & Sánchez, 2005), (Martínez, Liu & Yang, 2006), (Martínez, Liu, Yang & Herrera, 2005), (Zhang, Chen & Chong, 2004).

The choice of a domain of information to express preferences may be due to several reasons (Cabrerizo Lorite, 2008), (Sánchez Sánchez, 2007), (Chen, 2001), (Herrera & Herrera-Viedma, 2000), (Herrera & Martínez, 2001b), (Kacprzyk, 1986), (Levrat, Voisin, Bombardier & Bremont, 1997), (Martínez, 2007), (Xu, 2007):

- Experts with varying degrees of knowledge about the problem.
- Membership of experts, to different areas of knowledge.

- Quantitative or qualitative nature of the information with which it is working.

This is a topic studied by many researchers in the area of group decision-making. As a result, different approaches have been proposed to integrate different formats of representation of preferences (Chiclana, Herrera, & Herrera-Viedma, 1998), (Chiclana, Herrera, & Herrera-Viedma, 2001), (Chiclana, Herrera, & Herrera-Viedma, 2002), (Fan, Ma, Jiang, Sun, & Ma., 2006), (Fan, Xiao, & Hu, 2004), (Herrera, Martínez, & Sánchez, 2005), (Herrera-Viedma, Herrera, & Chiclana, 2002), (Herrera-Viedma, Martínez, Mata, & Chiclana, 2005), (Martínez, Liu, Ruan, & Yang, 2007), (Zhang, Chen, & Chong, 2004) y (Zhang, Chen, He, Ma, & Zhou, 2003), among others.

In addition, according to the literature (Arfi, 2005), (Delgado, Herrera, Herrera-Viedma & Martínez, 1998), (Fan, Ma & Zhang, 2002), experts used mainly three types of domains of information to express their preferences: numerical, intervalar and linguistic.

A summary of the main aspects of the different types of domains is presented. Special attention is given to the linguistic domain; this domain is very important in decision systems based on fuzzy logic, widely used and highly developed (Herrera, Martínez & Sánchez, 2005), (Kundu, 1997), (Zhang, Chen & Chong, 2004).

Numeric Domain

The use of the numeric domain in modeling preferences involves experts to express their preferences through numeric values. The main variants are the following:

- *Numeric binary*: It is characterized by using only two values {0, 1}, where 0 represents a negative assessment of the alternative and the 1 represents a positive evaluation. Example: The values given by the experts e_1 and e_2 are the following: U^1 = {1, 0, 0, 1}, the alternatives $x_1$and$x_4$are valued positively; U^2 = {0, 0, 1, 0}, the alternatives x_1, x_2 and x_4 receive a negative evaluation.
- *Numeric normalized in the interval [0, 1]*: The experts used a numeric value in the range [0, 1] for modeling the preference of each alternative (Fodor & Roubens, 1994), (Lee & O'Mahony, 2005). Example: Preferences given by the experts e_1 and e_2 are the following: U^1 = {1, 0.2, 0, 0.6}, the alternative x_1 is the best and assigned a maximum utility, consider the alternative x_3 worse than x_2 assigning a utility of 0 and 0.2 respectively; U^2 = {0, 0.4, 0.7, 0.9}, the best alternative would be x_4 and the worse x_1.

Intervalar Domain

The fact of considering the uncertainty in decision problems has led to the need to define models of preferences more flexible capable of collecting uncertainty, such as the intervalar modeling. The assessment of alternatives through intervals $[a_1, a_2]$ ($a_1 \leq a_2$) has been effective in decision problems (Alcalde, Burusco & Fuentes-Gonzalez, 2005), (Kundu, 1997), (Le Téno & Mareschal, 1998). In (Herrera, Martínez & Sánchez, 2005), (Kundu, 1998) experts express their preferences through the intervals [0, 1]. In the case that the intervals are not defined within this range it would only need to apply a normalization process in [0, 1].

Example: 1 and 2 experts express their preferences using an intervalar domain of expression in [0, 1] and utility vectors as: $U^1 = \{[0.5, 0.7], [0.2, 0.5], [0, 0.2], [0.7, 1]\}$, $U^2 = \{[0, 0.3], [0.3, 0.7], [0.7, 0.8], [0.8, 1]\}$, the best rated alternative valued by both experts is the 4, taking into account the ends of the intervals assigned to it.

Linguistic Domain

Experts can use a linguistic preference modeling (García-Lapresta, 2006)), (Herrera & Herrera-Viedma, 2000), (Tang & Zheng, 2006), (Turksen, 2007), (Zadeh, 1975), (Zadeh, 1996), (Sánchez Sánchez, 2007) in situations of decision in which the available information is too vague or they are valued aspects whose nature recommends valuations qualitative, above all if they are to evaluate aspects related to human perceptions often imprecisely expressed and where it is common to use the natural language words in place of numbers. As an example we can mention the proposed in (Levrat, Voisin, Bombardier & Bremont, 1997) to assess the level of comfort of a vehicle. In this case, the experts may prefer to use words like “bad”, “good”, “acceptable” to express their opinion on the level of comfort of a vehicle instead of numeric values.

Example: The preferences given by the experts 1 and 2 using utility vectors are as follows: $U^1 = \{$*very_bad*, *good*, *bad*, *very_good*$\}$, where the best valued alternative is x_4 and the worst rated is x_1; $U^2 = \{$*very_good*, *bad*, *very_bad*, *normal*$\}$, the best alternative is x_1 and the worst x_3.

The *fuzzy linguistic* approach (Zadeh, 1975) has been the discipline responsible for modeling the preferences of experts using linguistic assessments to express their preferences (Adamopoulos & Pappis, 1996), (Arfi, 2005), (Arfi, 2006), (Ben-Arieh & Zhifeng, 2006), (Bordogna & Pasi, 1993), (Delgado, Vendegay & Vila, 1992), (Herrera-Viedma et al., 2005), (Lu et al., 2007), (Ma, Ruan, Xu & Zhang, 2007), (Peláez & Doña, 2003), (Peláez et al., 2007), (Xu, 2004a), (Xu, 2006), (Zadeh, 1997), (Ekel & Silva, 2006).

Multi-granular linguistic domain

To work with multi-granular information, there is the model of fuzzy linguistic representation of 2-tuples (Herrera & Martinez, 2000), using tuples to represent linguistic information.

This model has been extended and applied in different process of aggregation of information on Decision Making Problems (DMP) (Herrera & Martínez, 2001a), (Peláez, Doña, La Red & Gil, 2009).

Linguistic information is represented by 2-tuples $(r_i, \alpha_i), r_i \epsilon S$ and $\alpha_i \epsilon$ [-0.5, 0.5), where S is the set of linguistic terms (label), r_i represents the center of linguistic label information and α_i is a numeric value which represents translation from the original result β to nearest label index in the set of linguistic terms (r_i), this is, the symbolic translation.

This linguistic representation model defines a set of functions to make transformations between linguistic terms, 2-tuples, and numeric values.

If $s_i \epsilon S$ is considered a linguistic term, then its equivalent representation in 2-tuples is obtained by means of the function θ as:

$$\theta: S \rightarrow (S \times [-0.5, 0.5 \quad)) \quad \theta(s_i) = (s_i, 0)/s_i \; \epsilon \; S$$

If it is considered that $\beta \epsilon [0, g]$ is a value that indicates the result of an operation of symbolic aggregation, then the 2-tuple which expresses the equivalent information to β is obtained with the following function:

$$\Delta: [0, g] \rightarrow S \times ([-0.5, 0.5 \quad))$$

$$\Delta(\beta) = \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \; \epsilon \; [-0.5, 0.5 \quad) \end{cases}$$

where s_i has the index of label closer to$\beta$and$\alpha$is the value of the symbolic translation.

There is always a function Δ^{-1}, that from a 2-tuple, returns its equivalent numerical value $\beta \epsilon [0, g]$:

$$\Delta^{-1}: S \times [-0.5, 0.5 \quad) \rightarrow [0, g] \Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Multi-granular information is represented by linguistic hierarchical structures, which allows to transform linguistic terms with different granularity of uncertainty and/or semantics, in a same domain of expression without any information loss. These linguistic structures allow improving the precision of multi-granular linguistic information aggregation processes.

A linguistic hierarchy is a set of levels, where each level is a set of linguistic terms with different granularity than the other levels of the hierarchy. Each level belonging to a linguistic hierarchy is represented by $L(t, n(t))$, where t is a number that indicates the level of the hierarchy, and $n(t)$ is the granularity of the linguistic terms set of level$t$. Belonging to a linguistic hierarchy levels are ordered according to their granularity.

Considering previous concepts, we will define a linguistic hierarchy (LH) as the union of all levels t:

$$LH = \bigcup_t L\left(t, n(t)\right)$$

To build a linguistic hierarchy, it is consider that the hierarchy is given by the increase in the granularity of the linguistic term sets in each level. Then the definition of S is extended to a set of linguistic terms, $S^{n(t)}$, where each set of terms belongs to a level of the hierarchy and has a granularity of uncertainty $n(t)$:

$$S^{n(t)} = \left\{S_0^{n(t)}, \dots, S_{n(t)-1}^{n(t)}\right\}$$

Generically, the linguistic terms of level set $t + 1$ is obtained from its predecessor as:

$$L\big(t, n(t)\big) \rightarrow L(t + 1, 2 \cdot n(t) - 1)$$

The main problem to aggregate multi-granular linguistic information is the loss of information produced in the process of normalization. To avoid this problem, hierarchical linguistic terms are used as multi-granular linguistic context. It is also necessary to use transformation functions between the hierarchy linguistic terms to make the processes of transformation without loss of information.

The transformation function of a linguistic label in level t to a label in the level $t + 1$, which satisfies the basic rules of the linguistic hierarchy, is defined as:

$$TF_{t'}^{t}\left(S_i^{n(t)}, \alpha^{n(t)}\right) = \Delta\left(\frac{\Delta^{-1}\left(S_i^{n(t)}, \alpha^{n(t)}\right) \cdot (n(t') - 1)}{n(t) - 1}\right)$$

The combination of 2-tuples and linguistic hierarchy allows you to merge information without loss of information and at the same time working with different domains of expression.

The possibility of working with different domains of expression allows experts to use the linguistic labels set which seems the best suited to each of them; previously defined as multiple sets of linguistic labels of different sizes, with their respective semantic.

Conclusion

This article made a brief review on the problem of the expression of preferences in decision models; It provides abundant bibliographic references related to the main concepts.

The main structures used for the representation of preferences have been shown, main domains of expression of preferences have been indicated, and the main concepts and numerous references have been introduced in all cases.

Numerical and intervalar domains are especially appropriate for decision support systems based on traditional logic; the linguistic domain is especially useful in decision support systems based on fuzzy logic, which also use fuzzy aggregation operators.

It has been detailed in the domain of special linguistic expression, especially the multi-granular linguistic. This is especially useful when the expert decision makers use linguistic label of different granularity, which involves sets of labels of different cardinality. In these situations, it is essential to have a procedure of translation of labels; this procedure will allow to translate labels from a set to another set, without loss of information, using 2-tuples. In these situations, it is essential to have a translation of labels a set procedure to another without loss of information, using 2-tuples. This is fundamental for aggregation operators that they must work with linguistic labels of different sets with different granularity and cardinality. It is also essential to translate the results of aggregation to linguistic labels of each of the considered linguistic labels sets.

Summarizing the above, it can be said that there is a significant diversity of ways of expressing preferences; this allows them to use the data structures in the most appropriate way in each case and according to the model of decision making that is used.

Future Lines of Work

Taking into account the detailed above, there are a variety of ways of expressing preferences, but in literature referred to them generally there is not progress in the study of which one is the most appropriate from the perspective of the decision-maker. Therefore, it is necessary to conduct a study determining which model of expression of preferences is preferred by decision-makers.

Be studied especially the following:

- Which method of expression of preferences seems more appropriate, from the point of view of the expert decision-makers, for different types of problems of increasing complexity. The following methods are considered: i) peer comparison valuation; ii) direct assessment tuples, using linguistic labels.
- What say the expert decision makers after using the peer comparison method.
- What say the expert decision makers after using the tuple linguistic method of direct assessment.
- What conclusions have the expert decision makers after comparing both methods among themselves.

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