

# THE ANALYTICAL STUDY OF STRENGTHENING THE HORIZONTALLY CURVED STEEL BEAM WITH CFRP LAMINATES

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## Abstract

This research is devoted to study the non-linear three dimensional behavior of horizontally curved steel girder strengthened by CFRP laminates under static loads. A new computer program NFASAC (Nonlinear Finite Element Analysis of Steel-Adhesive-CFRP) has been written for this purpose utilizing previous programs. Verification of present model is done by compare the predicted load-deflection curves and shear stresses profile over the adhesive material with experimental data. The static parametric studies proved that the increment in the ultimate load due to strengthening in some cases reaches up to 39%, 54%, and 110% whenever the CFRP has thickness 1.4mm, 2mm, and 4mm respectively. While the increment in the load of elastic stage due to strengthen reaches up to 32%, 50% and 75% for thickness of CFRP 1.4mm, 2mm, and 4mm respectively. Also the laminate of CFRP can confine the horizontally curved steel beam from twisting. The reduction in angle of twist was 89% and reduction in lateral deformation was 90% in some cases. The results showed that length of CFRP equal to 67 percent of length of span was adequate to achieve maximum strengthening. On the other hand the relation between thickness of CFRP laminate and increasing in ultimate load was linear. A governing equation was derivative to predict the increment in ultimate load of horizontally steel curved beam strengthening the bottom flange by CFRP laminate.

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**Keywords:** Steel, CFRP, Strengthening

## Introduction

Horizontally curved steel beams are frequently used in heavier structures such as composite girder bridges. In many cases these bridges are located in on- and off - ramps with various radii of curvature and are

characterized by complex vertical and horizontal geometries. As in any metallic structures the horizontally curved steel girders are deteriorating with time, thus the rehabilitation and strengthen of such structures become necessary.

### **Previous Researches**

The original Guide Specification for horizontally curved highway bridges was first officially published by AASHTO in 1980. (AASHTO, 2005) the LRFD specifications included an integrated design approach to curved girders (Fisher et al., 2006). (Hasebe et al., 1982) studied the effects of curvature and the effects of cross section dimensions on the effective width of horizontally simply supported curved beams of box and channel cross section under uniformly distributed loads. The authors concluded that the effective width ratio is independent on the cross sectional dimensions of the curved beam under uniform loading but it is dependent for concentrated loading. (Shanmugham et al,1995) tested ten steel curved beams. The results obtained from experiments on two sets of curved I-beams. The aim of this paper is to determine the ultimate load carrying capacity of steel I-beams with intermediate lateral restraint and to examine the effect of curvature on the behavior of these beams under bending loads. (Sennah et al. 1998) summarized the results from an extensive parametric study, using the finite element method in which 120 simply supported curved bridge were analyzed to evaluate the shear distribution in the web due to truck loading as well as dead load. (Young-Lin, 2001) and (Bradford, 2001) investigated the nonlinear elastic-plastic behavior of steel I-section beams curved in plan under vertical loading. The material and geometric nonlinearity are included in this study. Three types of simply supported I-section steel curved beam (continuously braced, centrally braced, and unbraced) under mid-span concentrated load were investigated analytically. Comparisons with rational finite element results and existing experimental results show that the proposed interaction equations give a good lower bound for the design of horizontally curved beams (steel I-section). (Matta et al, 2005) tested in tension continuous reinforcements and double shear lap joints. The experimental results showed that stiffness significantly increased in bonded specimens and that specimens failed at a monotonic load 12% higher than that associated with the steel yielding. Failure occurred at the steel-adhesive interface due to the CFRP plate debonding. A strain saving of 20% was also observed in the steel member. Finally, analytical models to characterize the load transfer mechanism were validated. (Al-Mutairee, 2008) presented a computer program called NFHCBSL to analyze a horizontally curved beams (steel, concrete and composite) subjected to static loads. His investigation was proved that the plastic zone initiated under the concentrated load at

extreme web edge of the simply supported horizontally curved beam and the effect of the non-prismatic included. (Kirk et al., 2009) instigate the behavior of straight steel beam strengthen with high modulus carbon fiber reinforced polymers CFRP product (sheet and laminate) using different epoxy, also study the bond and development length of CFRP, the results of strengthening show increase in ultimate load up to 32% compare with unstrengthening beam when using two layered of CFRP laminate. (Al-Mutairee, 2013) studied the effects of strengthening the bottom flange of steel I-section simply supported horizontally curved beams on the behavior and maximum strength of these beams under concentrated static load. The strengthening includes increasing the thickness of bottom flange at finite region (angle of strengthening, ( $\varphi$ )), thus the volume of steel will be increased. The results show that the ultimate strength of beam can be increased about 35%, while the maximum vertical and the horizontal deflections can be reduced by 67% and 69% respectively.

### **Research Objective**

All previous studies did not investigate the effects of strengthening horizontally curved beams by CFRP on the behavior or ultimate strength of these beams. Therefore, the main object of this research is to study this effect which is important to improve the strength and performance of steel I-section horizontally curved beam.

### **Nonlinear Finite Element Program (NFASAC)**

The main objective of the NFASAC program was to analyze non-linear composite steel-adhesive-CFRP members under general three-dimensional states of loading. The program is coded in FORTRAN language. In this study, the computer program NFASAC has been written to be suitable for analyzing different cases and to compare the analytical results with available experimental data in order to examine the applicability of the computer program in the analysis of such cases. The main features of the program are listed below.

### **Steel Representation**

The 20-noded isoparametric brick element has been used in the program to represent different type of element material such as steel element, and imaginary brick element, each node have three degree of freedom.

### **Shear Interface (Adhesive Material Representation)**

To modeling the behavior of debonding phenomena into interface (adhesive) shear connectors are used for this purpose. Shear connectors are simulated by a space bar element which has two nodes with three

translational degrees of freedom at each end. The degrees of freedom are the same as those used in a normal beam except that no rotation freedom is considered.

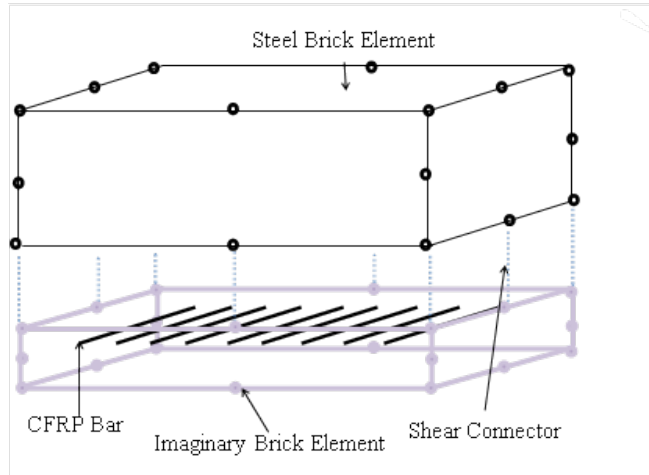


Figure (1): Formulation of Interface Element and CFRP Product in the Presented Program (NFASAC).

### CFRP Representation

CFRP is simulated as reinforcing bars (axial members) embedded in to imaginary 20-node brick elements.

### Integration rule used

For evaluation of the stiffness matrices of 20-node brick elements, the 27 (3x3x3) Gauss-quadrature rule has been applied in the computer program. While two Gauss points are used to calculate the stiffness matrices of reinforcing bars.

### Loading schemes

The computer program employs a load increment scheme with the size of loading increment being specified by the user. In finite element analyses, the external concentrated loads are simulated by a set of equivalent nodal forces.

### Convergence criteria

The force and displacement convergence criteria are available in the program.

### Continuity analysis after debonding or rupture

The program can be achieved all stages of behavior of debonding or rupture phenomena. The continuity after debonding or/and rupture is

performed by redistributed internal stress for whole structure. The redistributed of internal stress achieved through re-run the analysis with last condition (degradation state) and calculate new stiffness according to last degradation state.

**Reliability of the Program**

In order to verify the reliability of the computer present model NFASAC, the following example is considered here between NFASAC and experimental data (Kirk, 2009) for straight steel beam strengthen with CFRP was done.

Super Light Beams, SLB100 x 4.8, were used, with an additional 6.5 mm (1/4 inch) thick steel plate that was stitch welded to the compression flange along its length to simulate the concrete deck and consequently shift the neutral axis towards the compression flange approximately 10 mm higher than center of the steel beam alone (Kirk, 2009). The length of the beam is 864 mm and the clear span is 813mm, section details as shown in figure (2). The beams loaded under four-point loading using a spherically seated bearing block as shown in figure (3).

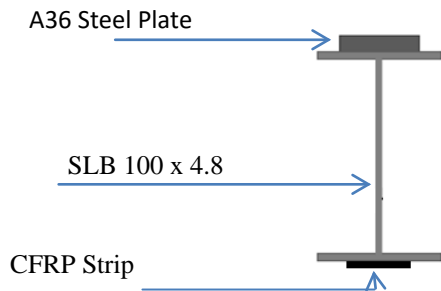


Figure (2): Section Details of Kirk Beam (Kirk, 2009).

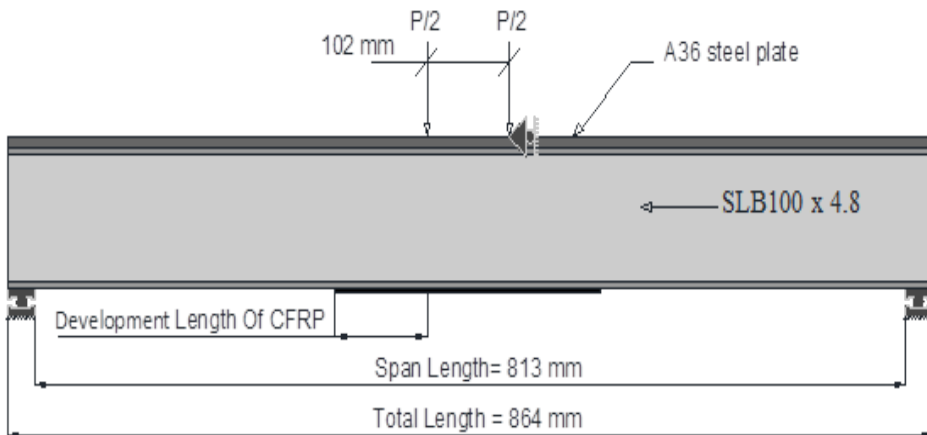


Figure (3): Geometry, Properties, and Loading Conditions of the Kirk Beam (Kirk, 2009).

From the results of the convergence study on mid span deflection, it was found that model with 544 elements (3853 nodes) shown in figure (4) can be used, where the difference between the last two meshes is equal to 0.8%. Furthermore the total length of CFRP Laminate was 306 mm. While the number of imaginary brick element, number of bar in cross section, and number of studs were 60, 11, and 215 respectively.

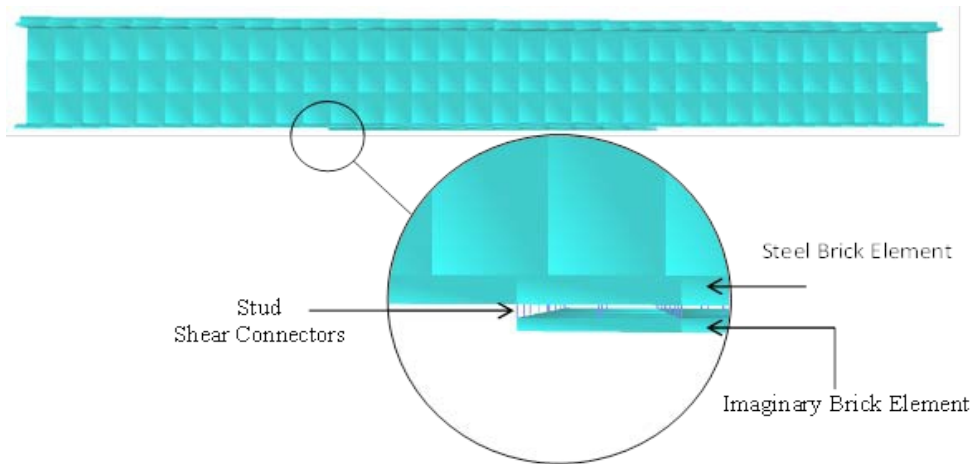


Figure (4): Finite Element Mesh of 544 Brick Elements of Kirk Beam.

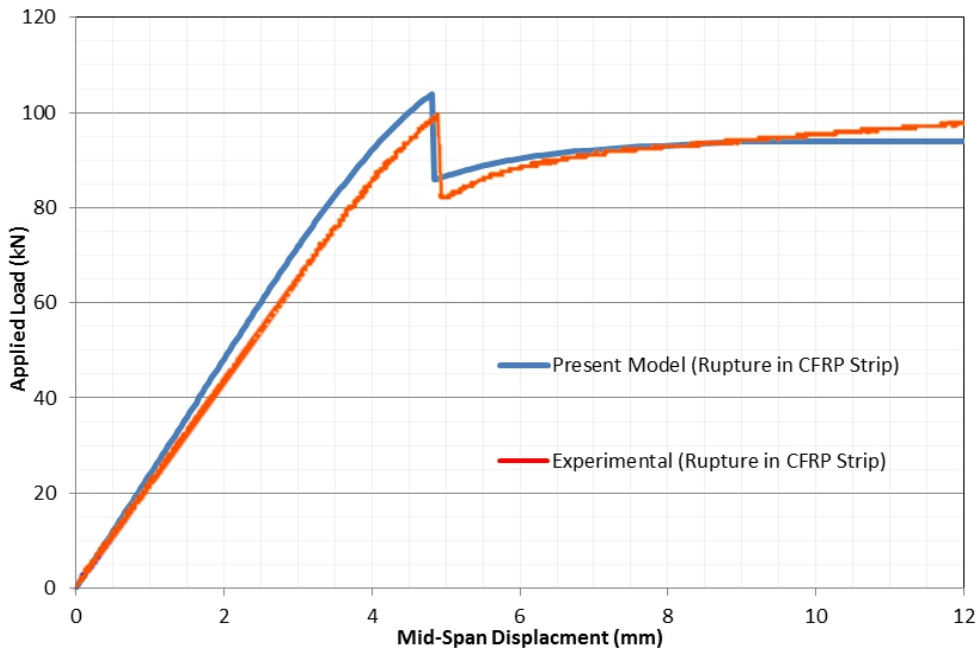


Figure (5): Predicted Load-Deflection Curve for Kirk Beam Strengthened by CFRP Laminate.

The results showed good agreement between the present model and the experimental for modeling debonding phenomena and rupture of CFRP as shown in figure (5). The maximum deference in ultimate load between NFASAC results and experimental data about 5% at CFRP rupture.

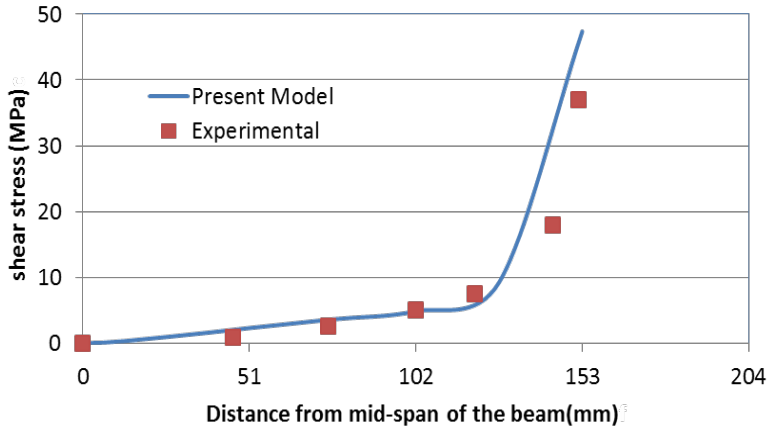


Figure (6): Comparison between Predicted Shear Profile at Adhesive (At Lower Face of Bottom Flange) with the Experimental Results for Kirk Beam Strengthened with CFRP.

The comparison between predicted shear profile and experimental results as shown in figure (6), the results show good agreement and confidence on validity of the present model.

### Parametric Study

Several parametric studies have been carried out using present model NFASAC to investigate the influence factors which are concerned with benefit of using CFRP material with horizontally curved steel beam. These parameters are listed as follows:

- 1- Effect of the location of CFRP laminate in cross section on the behavior of horizontally curved steel beam.
- 2- Effect of the length of the span of the strengthened horizontally curved steel beam to the depth of section (L/D ratio). Taken three values (12, 18, 24)
- 3- Effect of curvature ( $\theta$ ) on the behavior of strengthened horizontally curved steel beam. The studies values (5, 25, 45)
- 4- Effective length of the CFRP laminate (LC ratio). (LC= length of CFRP laminate to the length of span). Three values had been investigated (1/3, 2/3, 1)
- 5- The effect of thickness of CFRP laminate according to the ultimate load.

The curved beam was simply supported with bracing against lateral torsional buckling at four points along length of beam as shown in figure (8). The type of load was distributed load over the equivalent deck area, the distributed load simulated as nodal load act on the upper nodes of equivalent deck. The details of cross section for parametric studies as shown in figure (7) below. And the locations of CFRP laminates were detailed as shown in figure (9).

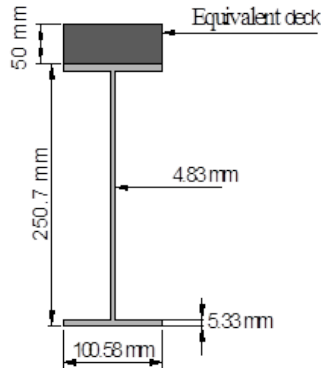


Figure (7): The Details of W10x12 Section Composite with 50 mm Equivalent Deck.

The material properties for steel section and equivalent deck is constant for all parametric study, it's assumed to be 200 GPa, 350 MPa and 0.3 for modulus of elasticity, yield strength and Poisson ratio respectively. And the angular length (L) of span calculated according to L/D ratio. Radius of curved determined according to L and curvature parameter ( $\theta$ ).

The CFRP material properties for the parametric study selected from BASF Company, which is developed an ultra-high modulus laminate (known as MBrace laminate) with tensile modulus (E) of 450 GPa, tensile Strength ( $F_u$ ) of 1500MPa, strain rupture ( $\epsilon_u$ ) is equal to 0.33%, the thickness and width of laminate are equal to 1.4 mm, 50mm respectively.

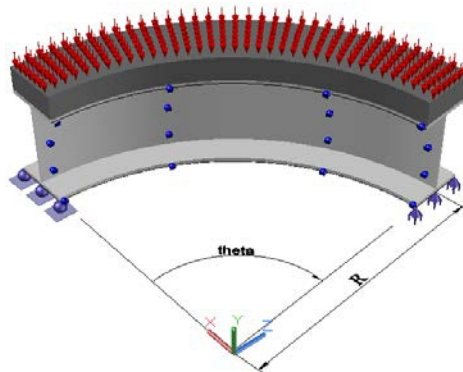


Figure (8): Boundary Condition for Horizontally Curved Steel Beam Used in the Parametric Studied.



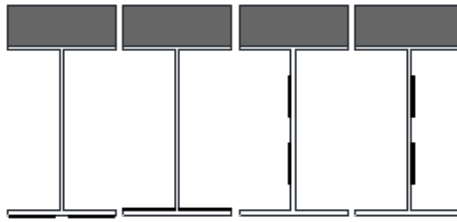


Figure (9): Location of CFRP Laminate on Cross Section of the Steel.

**Results of Parametric Studies**

To avoid confusion in reading results, all beams solved here were named according to unique numbering system. the first number refers to (L/D) ratio, second number refers to curvature ( $\theta$ ), third number refers to (LC) ratio, and the last refers to location of CFRP laminate (e.g. beam 3124 have span length to depth (L/D) ratio=24, theta=5, Length of CFRP laminate to the length of span (LC) ratio=2/3, and the laminate located outside the face of web) .The unique numbering system was detailed in table (1).

Table (1): Numbering system for all beam solved in chapter five

Parameter Number	(L/D) Ratio	$\theta$ (deg.)	LC Ratio	Location of CFRP
0	-	-	0	Unstrengthening
1	12	5	1/3	Lower face of bottom flange
2	18	25	2/3	Upper face of bottom flange
3	24	45	1	Inside face of web
4	-	-	0	Outside face of web

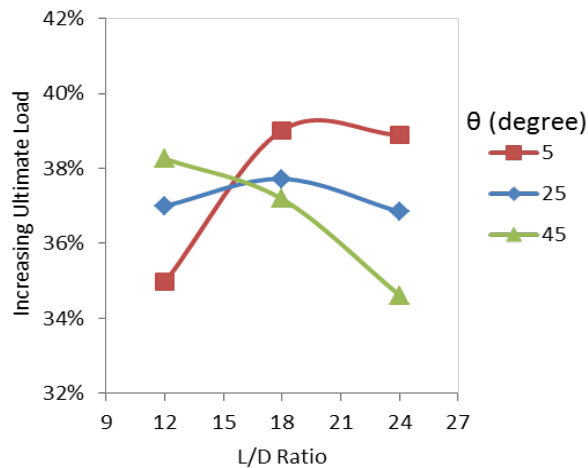


Figure (10): Effect of (L/D) Ratio on Increment in Ultimate Load of Horizontally Curved Steel Beam Strengthened by CFRP (Strengthening Lower Face of Bottom Flange, LC=100%, Thickness of CFRP=1.4mm).

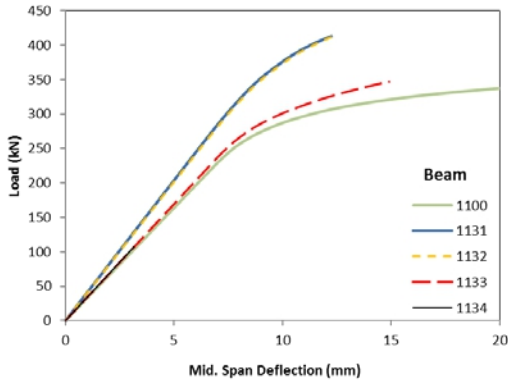


Figure (a) Beams have L/D=12,  $\theta=5$ , LC=1

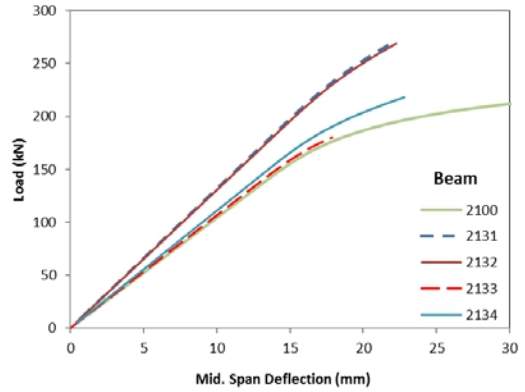


Figure (d) Beams have L/D=18,  $\theta=5$ , LC=1

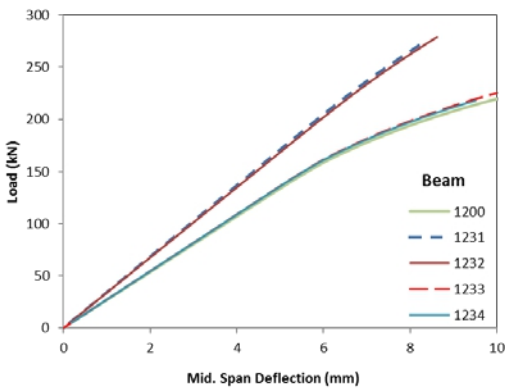


Figure (b) Beams have L/D=12,  $\theta=25$ , LC=1

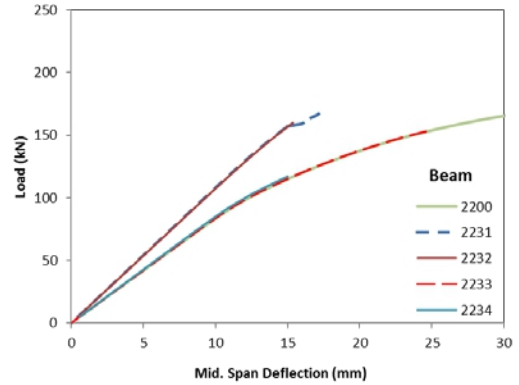


Figure (e) Beams have L/D=18,  $\theta=25$ , LC=1

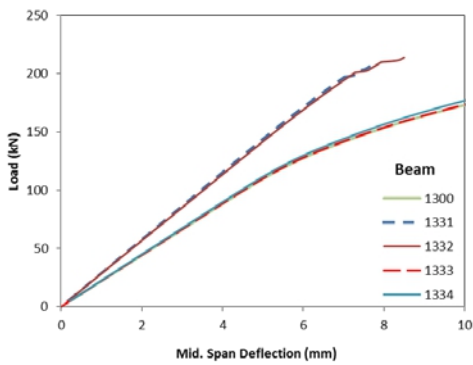


Figure (c) Beams have L/D=12,  $\theta=45$ , LC=1

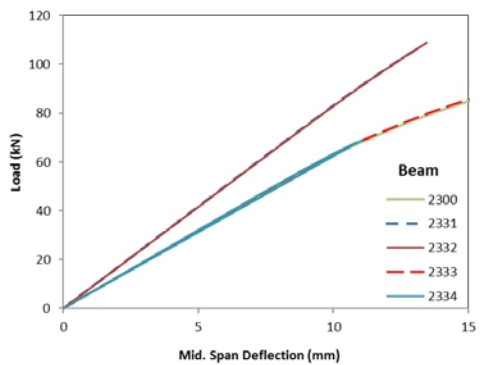


Figure (f) Beams have L/D=18,  $\theta=45$ , LC=1

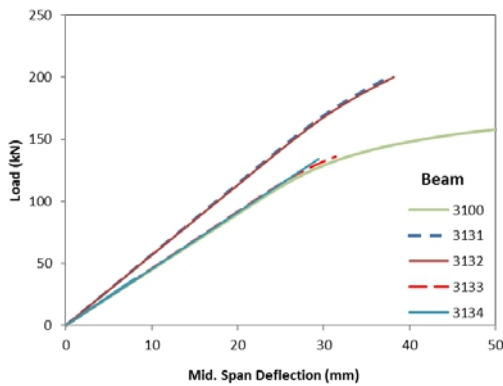


Figure (g) Beams have  $L/D=24, \theta=5, LC=1$

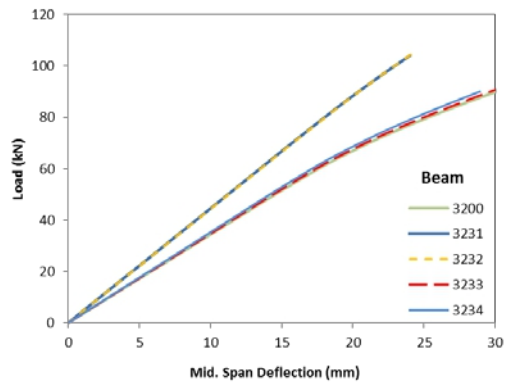


Figure (h) Beams have  $L/D=24, \theta=25, LC=1$

Figure (i) Beams have  $L/D=24, \theta=45, LC=1$

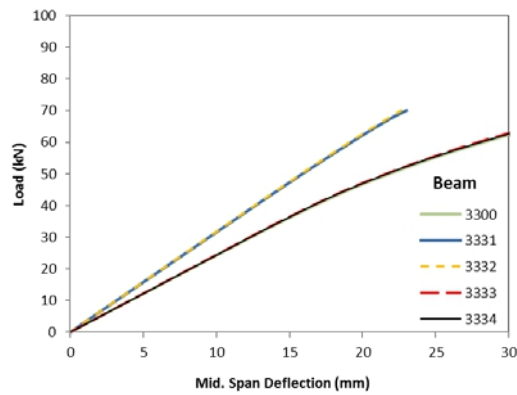
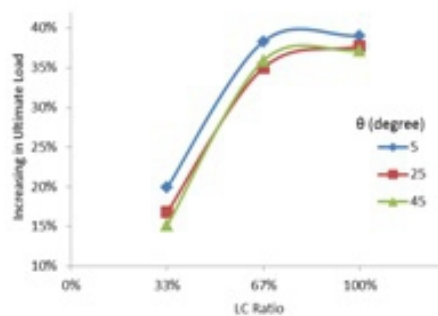
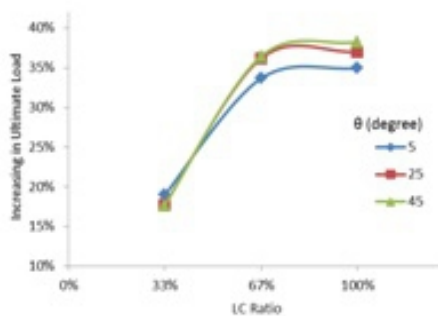
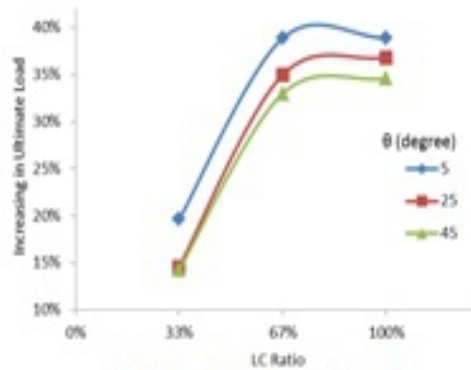


Figure (11): Load-Deflection Curve for Different  $L/D$  and  $\theta$  to Explain the Optimal Location of CFRP Laminate (thickness of CFRP=1.4mm).



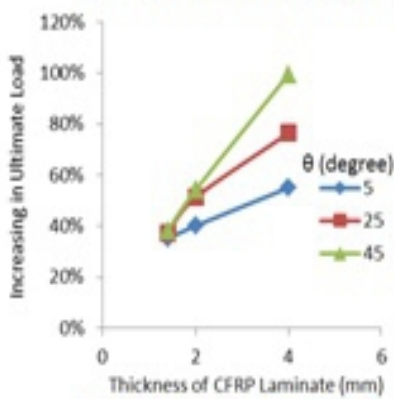
(a)- For L/D Equal to 12.



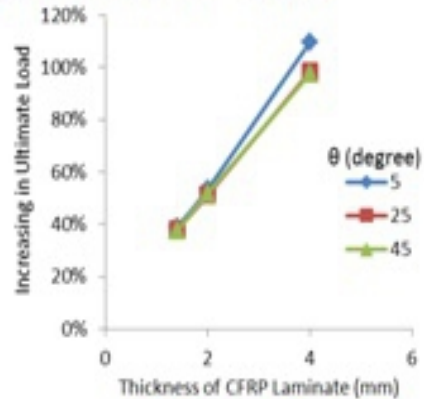
(b)- For L/D Equal to 18.

(c)- For L/D Equal to 24.

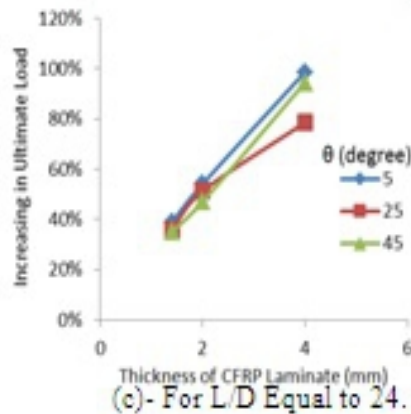
Figure (12): Increment in the Ultimate Load Due to Strengthening (Located in Lower Face of Bottom Flange, thickness of CFRP=1.4mm).



(a)- For L/D Equal to 12.



(b)- For L/D Equal to 18.



(c)- For L/D Equal to 24.

Figure (13): Effect of Thickness of CFRP on Increasing in Ultimate Load (LC=1, CFRP Laminate in Lower Face of Bottom Flange)

Table (2): Increment% in ultimate load due to strengthening.

		L/D =12			L/D =18			L/D =24						
		t			t			t						
		1.4	2	4				1.4	2	4				
θ	5	35	40	55	θ	5	39	53	110	θ	5	39	54	99
	25	37	51	76		25	38	51	98		25	36	51	79
	45	38	54	99		45	38	52	98		45	35	47	94

Where t is the thickness of CFRP laminate.

### Discussion of Results

As shown in figure (11) for all angles of curvature, and L/D ratio, it’s found that the optimal location of CFRP was at the bottom flange. The varying behavior when change location of CFRP is due to distribution of stresses in I-section, because the maximum normal stresses exist in lower bottom flange and decrease whenever reach the neutral axis.

The results showed that the increment in ultimate load was increased when L/D ratio increasing for curvature (θ) less than or equal to 25 degree and for L/D less than 18, but for curvature (θ) greater than 25 degree the increment in ultimate load was decreasing when curvature’s increases (θ). The results showed that the increment in ultimate load is increased with increasing (θ) for L/D less than 16, but for L/D greater than 16 the increment in ultimate load is decreased with the curvature’s increasing (θ).

For elastic stage the increment in ultimate load is increased with increasing (θ) for all L/D ratios.

As shown in figure (10) there is an interaction between L/D ratio and curvature of beam, this interaction controlled the type of failure of curved beam, and influence directly on benefit of strengthening with CFRP.

### Derivative of Increment Equation

The increment of ultimate load due to strengthening for cases between the parameter (L/D=12-24, θ=5-45, thickness of CFRP=1.4-4) can be interpolate by equation (4). This equation derivative statistically from numerical data which result from finite element analysis. Thus, the equation applicable for the cases have the same limitations and conditions of numerical data. The numerical data of increment in ultimate load was summarized table (2).

Assumed the increment in ultimate load due to strengthening for specific L/D ratio as following:

$$Increasing = a1 + a2 * t + a3 * \theta + a4 * t * \theta \quad \dots Eq.(1)$$

Where  $t$  : thickness of CFRP,  $\theta$ : angle of curve,  $\{a_1, a_2 \dots a_4\}$  : constant. By apply boundary conditions from table (2) in equation (1) the vector  $\{a\}$  can be written as:

Table(3): The values of constants of increment equation (vector  $\{a\}$ ).

	L/D=12	L/D=18	L/D=24
a1	0.267047	6E-04	0.071866576
a2	0.056867	0.278	0.229916726
a3	-0.00475	0.001	-0.000932267
a4	0.003961	0	-2.8539E-05

To generalize the equation (1) for any L/D ratio, the constant  $\{a\}$  must be relation with L/D ratio. Thus, the equation (2) assumed to represent this relation.

$$a_i = b_{i1} + b_{i2} * (L/D) + b_{i3} * (L/D)^2 \quad \dots \text{Eq.}(2)$$

Where  $a_i$ : represent the parameter  $a_1, a_2 \dots a_4$ .

$b_{ij}$ : constant

The vector  $\{b\}$  can be calculated by applied the boundary conditions from table (3) in equation (2), the results:

$$\{b\} = \begin{bmatrix} 1.81308995330 & -0.18512278210 & 0.00469049200 \\ -1.19317321510 & 0.14904468220 & -0.00373955280 \\ -0.04024293940 & 0.00427785440 & -0.00010999620 \\ 0.03196169580 & -0.00333382410 & 0.00008337070 \end{bmatrix} \quad \dots \text{Eq.}(3)$$

By substitute the equation (2) in (1), yield:

$$Increasing = [1 \quad t \quad \theta \quad (t \cdot \theta)] \begin{bmatrix} b_{12} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} \begin{bmatrix} 1 \\ L/D \\ (L/D)^2 \end{bmatrix} \quad \dots \text{Eq.}(4)$$

The R squared measure of goodness of fit of equation (4) equal to 0.98.

**Conclusion**

Based on the analytical results obtained from 144 examples in the present study, several conclusions can be drawn and summarized as follows:

- 1- The benefit of strengthening the steel material with CFRP is related to the properties of CFRP products and steel material. When used the NM (Normal modulus with high tensile strength) of CFRP, the increasing in load-deflection curve occurred after the steel yielding. Here the CFRP will be compensating the yielded region of steel, while when used the UHM (Ultra high modulus with moderate tensile strength) of CFRP, the increasing in load-deflection appear in elastic stage clearly.

- 2- The optimal location of CFRP products in I-section horizontally curved steel beam was found in the bottom flange. No feasible increasing observed when strengthens the web along the direction of span.
- 3- For angle of curvature ( $\theta$ ) less than or equal to 25 deg. and L/D less than 18 when the L/D increased, the increasing in ultimate load due to strengthening will be increased. For angle of curvature ( $\theta$ ) greater than 25 deg. and when the L/D increased, the increasing in ultimate load due to strengthening will be decreased.
- 4- For L/D ratio less than 16 and angle of curvature ( $\theta$ ) increased, the increasing in ultimate load due to strengthening will be increased. For L/D ratio greater than 16 and angle of curvature ( $\theta$ ) increased, the increasing in ultimate load due to strengthening will be decreased.
- 5- The UHM CFRP Laminate can reduce the twisting of the horizontally curved steel beam. The reduction in angle of twist was 89% due to strengthening. Also the lateral deformation is confined due to strengthening, the reduction in lateral deformation was 90%.
- 6- Maximum increment observed in the ultimate load was 39%, 54%, and 110% for beam 2131 strengthening with CFRP has thickness 1.4mm, 2mm, and 4mm respectively.
- 7- The present finite element model (NFASAC) used in the present study is able to simulate the behavior of horizontally curved steel girders strengthened by CFRP, and able to modeling the debonding phenomena. Furthermore, NFASAC able to simulate the straight beam strengthened by CFRP, the predicted ultimate load-deflection curves and shear profile of adhesive are in good agreement with the experimental data, maximum different between them was 5%.

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