Demand Uncertainty Reduces Market Power and Enhances Welfare

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Abstract

Classical welfare economics assumes that the demand function, or consumers' utility, is known with certainty. Probabilistic microeconomics generalizes it by maximizing expected utility, or by optimizing under a specific constraint. Existing research has provided only limited insight into the welfare effects of demand uncertainty, and that limited insight suggests welfare reduction as a result of demand uncertainty. In contrast with previous works, our paper does not prescribe the form of demand uncertainty, but rather derive it from individual consumers' choices. We then analyze monopolist optimization problem, first constrained by a "Safety-First" type condition imposed on the coefficient of variation, and then by considering risk-adjusted profit measure. Our results indicate that the Marshallian welfare measure, when compared with the deterministic model, increases with uncertainty of the demand function.

We point out that uncertainty characterizes markets that lie between the pure monopoly model, and perfect competition model. We believe that our model of demand uncertainty is a realistic one, very much like observed behavior of markets. Most importantly, our work suggests that transition from monopolistic market structure to competitive one may be explained better by demand uncertainty than by mere presence of competitors, as opposed to the instant appearance of competitive pricing in common textbook models.

Finally, we show how a demand can be efficiently estimated from simple consumer surveys (admitting its random structure).

Keywords: Demand uncertainty; welfare; market power

Introduction

Neither a pure monopoly nor a pure competition is a realistic description of markets for goods and services, which we observe in real life. One possible alternative to those two is the *monopolistic competition,* in which each producer holds a monopoly for a product, with those products being close substitutes. This phenomenon is indeed more common in the real world than in economics textbooks: consumers hardly ever face a choice between two (or more) identical products or services. Even two gas stations selling gasoline across the street from each other differentiate their products with brand names, additional services, or ethanol content.

Things in the real world are even more complicated. In reality, a consumer does not decide to make a purchase of a good or service in a purely deterministic fashion, but rather each individual purchase decision is quite uncertain, and affected by numerous factors, such as preferences of the consumer, the price, current mood or confidence of the consumer, availability of substitutes, etc. If the purchase decision were indeed fully determined by the purchase price, as the standard deterministic models suggest, then massive advertising campaigns developed by businesses would appear quite impractical. In reality, advertising, as well as sales, coupons, etc., work, as they all may affect the consumer in the moment of the purchase decision. They are only in the limited degree functioning as the source of information about price and quantity, and to a much larger extent they seek to affect the momentary lapse of judgment, or momentary depth of it, at the moment of purchase.

For the two idealized market models present in economics textbooks, monopoly and perfect competition, the above argument does not apply. In the pure monopoly model, the consumer faces a price given by the monopolist, and other factors are generally fully set by the aggregate market structure. The purchase decision is basically determined by the price. Paradoxically, perfect competition produces a similar "no uncertainty" outcome, as the price is effectively set by the relationship of the industry cost structure and the industry demand, and again each individual consumer's purchase decision is basically determined by the price.

It is the "grey area" of markets between perfect competition and pure monopoly that result in greatest uncertainty of the consumers' purchase decisions. Consumers can always change their minds and go to another seller, even if the price is higher there, because of receiving a slightly differentiated product that they are happier with, or just not feeling like buying now, but getting that magic shopping feeling later. They may be affected by advertising, reputation, promotions, etc.

Because of their welfare implications, monopolistic markets are generally perceived as socially undesirable, while perfectly competitive

markets are viewed as socially desirable. Realistic markets seem to be viewed as a messy "in-between" case. But even this perception begs a question: are realistic markets in equilibrium of their own, or do they evolve over time towards one of the two purely theoretical concepts? In this work, we create a model that seems to indicate that realistic models naturally converge towards competitive model as a result of uncertainty, and that uncertainty of demand may in fact result in increased welfare.

Modeling uncertainty of demand

Classical welfare economics is developed under the assumption of complete knowledge of the demand function. In reality, producers face great uncertainty in assessing consumers' demand, even in a monopolistic or perfectly competitive market. The aggregate demand is downward sloping both in the case of monopolistic and competitive markets, and in the real business world, the actual demand is subject to both measurement uncertainty, and the uncertainty of the underlying functional relationship. The effects of randomness of demand has been studied, generally, with the objective of expected utility maximization, or under a "Safety-First" constraint, with some theoretically prescribed random structure of demand. Our work proposes what we believe to be a more practical alternative.

McCall (1971) provided an early survey of probabilistic microeconomics. He stated: "work is underway in reformulating the theory of the firm under conditions of uncertainty, in assessing the role of information in general equilibrium theory, in integrating uncertainty with welfare economics, and in designing measures of uncertainty possessing both theoretical appeal and practical importance." While the use of expected utility maximization is most common, Hanoch and Levy (1969) provided examples of an situations with lower expected value and higher variance having higher expected utility, suggesting that maximization of expected utility may produce counterintuitive results. Brown G., Jr. & Johnson, M.B. (1969), and in a comment to their work, Visscher, M. L. (1973), discussed the effect of demand uncertainty in a limited case of linear demand, with their results indicating that uncertainty of demand may result in welfare improvement. In contrast to that work, Leland (1972) used the expected utility models to study the output (and by implication, welfare) effect of uncertainty on a monopolist facing a random demand. In particular, all of Leland's results indicate that uncertainty cause reduction in monopolist's output. Leland (1972) nevertheless concluded that welfare effects of models studied by him remain unclear. Dana (1999) analyzed firms facing uncertain demand under constraints of costly capacity and prices set in advance, and concluded that such firms will sell their output at multiple prices, i.e., will utilize price discrimination. Equilibrium price dispersion with homogeneous

277

goods was first described by Prescott (1975) and developed more formally by Eden (1990). Unlike the approaches of Dana (1999), Prescott (1975) and Eden (1990), our model precludes price discrimination and assumes uniform pricing. We also do not prescribe stochastic demand structure, but rather derive it from consumers' preferences. Also, in contrast to the results of Day, Aigner and Smith (1971), our model suggests that demand uncertainty will not produce safety margins in pricing, but rather lead to lower prices.

Roy (1952) provided an investment perspective on an alternative optimization approach, where the maximization of profit, or utility of profit, is constrained by the condition that the losses cannot exceed certain level with prescribed probability. This approach has been given a general name of *Safety-First.* Haim and Sarnat (1972) point out that Roy's Safety-First methodology can be reduced to the expected utility approach. Day, Aigner and Smith (1971) review Safety-First, in the form of three rules for a profit-maximizing monopolist. They find that under *Strict Safety-First,* where the risk condition is binding for optimization, output exceeds the output under the other two, softer forms of Safety-First. Day, Morley and Smith (1974) argue that a model in which the company's financial position affects its investment opportunities may provide an explanation as to why risk constraints are important in a monopolist's pricing decision.

We believe that our approach provides an alternative to the above models. We propose that the random demand can be derived from individual consumers' choices, and that the optimization can be either performed with respect to expected utility, or constrained by a binding condition on the coefficient of variation of demand (and, equivalently, of company's profit). The resulting model shows that demand uncertainty actually expands output, thus providing potential for welfare enhancement. An additional bonus of our methodology is a new demand derivation procedure that can be used in practice for estimation of the demand function as well as of the marginal utility of the clients through simply designed consumer surveys.

A new model

As the existing literature indicates, demand uncertainty can be modeled in many ways. We propose a new model for it. We assume that the firm is a monopolist facing random demand, i.e., for every price, the quantity demanded is a random variable $Q(p)$, with $E(Q(p))$ denoted by $q(p)$ and $\text{Var}(Q(p))$ denoted by $v^2(p)$. Mathematically, this means that the demand function is a stochastic process $\{Q(p)\}_{p \geq 0}$. In our model, the demand is derived as a sum of individual Bernoulli Trials: decisions about purchase. Each purchase decision can be viewed as a comparison of the externally given price with individual's preference for the item purchased, or as a utility

comparison. In a standard additive utility model, if one alternative is a *numeraire* good and the utility of the other alternative, a good or service purchased, is $U(q)$, where q is the quantity, the purchase decision is made if marginal utility $\dfrac{dU}{dq}$ exceeds the price of additional item purchased. This decision is equivalent to comparing the reservation price, a random variable in our model, to the market price p. We propose that either the marginal utility or the reservation price can, and should be, treated as a random variable, to properly account for the uncertainty of each purchase by an individual consumer. This uncertainty has been only marginally acknowledged in theoretical economic literature, while it is overwhelmingly acknowledged by the massive advertisement expenditures of businesses, which perceive consumers' preferences as uncertain and, most importantly, changeable.

Consider a firm that has *N* potential customers in a certain region. If this firm lowers its price, not only will more consumers from its region purchase the firm's product, but also consumers from other regions may arrive. In our model, we will therefore assume that *N* is a decreasing function of *p*. Let us note that we do not assume that the *N* consumers all purchase the product, but rather that they all have some probability of purchasing it, i.e., are potential customers.

So let us assume that there are *N(p)* independent potential consumers in the market, and that each of them buys the product under consideration independently with probability $\theta(p)$, where $0 \le \theta(p) \le 1$, a function of *p*, with the following properties: $\theta(0)=1$, $\lim_{q \to \infty} \theta(p)=0$, and $\theta(p)$ is strictly decreasing. This means effectively that each consumer's purchase is an independent Bernoulli Trial (Fabian and Hannan, 1985). Under these assumptions, the total demand in time horizon *T*, given a price *p*, is a random variable $Q(p)$ with a distribution described by:

$$\Pr(Q=k)=\binom{N(p)}{k}\left(\theta(p)\right)^{k}\left(1-\theta(p)\right)^{N-k}$$

for $k=0,1,...,N(p)$. Then $q(p)=E\left(Q(p)\right)=N(p)\theta(p)$ is the expected value of the output, and its standard deviation is

$$v(p)=\sqrt{N(p)\theta(p)\left(1-\theta(p)\right)}.$$

The firm serving random demand cannot maximize profit without regard to uncertainty of the demand. One possible approach, most commonly used, is to decrease the expected profit by some measure of risk. Another

form of risk-adjustment is a "Safety-First"-type condition. We will begin with that second approach.

Safety-First Approach. The objective is to maximize expected profit while controlling risk, in a given time horizon T. The overall uncertainty may increase because of an increase in the number of potential customers as well as increased uncertainty of individual purchase. We will analyze risk per unit purchased. What is then a rational profit strategy? Assume for simplicity that there are no fixed costs, and the marginal cost is constant and equal to c. The strategy we propose is:

$$\text{Maximize} \quad E(Q(p)(p-c)),$$

$$\text{subject to} \quad \frac{v(p)}{q(p)} \le \alpha. \tag{1}$$

The quantity maximized can be also written as $(p-c)q(p)$. The constraint states that standard deviation of price per quantity unit (i.e., coefficient of variation of demand or, equivalently, of profit) is bounded by a certain parameter α. This is a form of a strict Safety–First condition, although it is somewhat different than the one used by Day, Aigner and Smith (1971).

We will now derive a more explicit formulation of the maximization problem (1). The expected profit, $\Pi(p)$, is given by

$$\Pi(p) = N(p)(p-c)\theta(p).$$

The optimization problem becomes:

$$\text{Maximize} \quad N(p)(p-c)\theta(p),$$

$$\text{subject to} \quad \frac{1}{\sqrt{N(p)}}\sqrt{\frac{1-\theta(p)}{\theta(p)}} \,\square\, \alpha.$$

Since the function $\theta(p)$ is strictly decreasing, the constraint can be also written as

$$p \le \theta^{-1}\left(\frac{1}{1+N(p)\alpha^2}\right). \tag{2}$$

Note that the inverse function θ^{-1} will also be strictly decreasing. To avoid a trivial situation, we assume also that c satisfies (2) (otherwise the set of admissible prices would be empty).

The producer's optimization problem, as stated above, has some interesting properties. First, as the number of potential customers approaches infinity, the expression $\dfrac{1}{1+N(p)\alpha^2}$ approaches zero, and the constraint condition is naturally satisfied. Thus for larger potential markets, we can

expect the constraint (2) to be more easily met. Furthermore, since θ and N are decreasing with p, reducing p will increase the firm's chance of meeting the risk constraint (2), even if the optimal price is not known to the firm. On the other hand, if the optimal price without risk constraint is expected to be high, then (2) may not be satisfied at that high price level. This means that a product with a relatively high expected profit per unit (in relation to the size of the market), will be sold below the price that would be optimal in the absence of risk, if the risk constraint is imposed and is binding.

   Risk-Adjusted Profit Approach. We will now present an alternative optimization approach. We propose a *risk-adjusted profit* measure, defined as:

$$\Pi_R(p) = \Pi(p) - \gamma \cdot N(p)\theta(p)(p-c)g(p) = \Pi(p)\theta(p)(p-c) \cdot \big(1 - \gamma g(p)\big)$$

   Here, γ is a coefficient of risk-aversion, and g is a measure of risk in demand uncertainty per unit of expected risk. We will assume that g is a differentiable and strictly increasing function on $(0, \infty)$. For technical reasons we will also assume that N and θ are differentiable functions. For example, if we measure the risk of profit by the standard deviation of the profit, then

$$g(p) = \sqrt{\frac{1 - \theta(p)}{N(p)\theta(p)}}.$$

   Since N is decreasing and θ is strictly decreasing in p, we conclude that so defined g is strictly increasing. We assume also that γ is such that $1 - \gamma g(p) \geq 0$ for some $p > c$ (otherwise the risk adjusted profit could not be positive for any $p \geq c$).

   The above models proposed here assume homogeneity of consumers. But our methodology can naturally be extended through aggregation of homogeneous submarkets to a heterogeneous market.

Output and welfare implications for a Safety-First Model

   Assume that the optimal price calculated in the absence of risk, without the risk controlling constraint (2), exists and denote it by p^*. A key assertion of this section is

If $p^* > \theta^{-1}\left(\dfrac{1}{1 + N\alpha^2}\right)$ then the optimal choice might be either the solution to

the equation $\theta(p) \cdot (1 + N(p) \cdot \alpha^2) = 1$, denoted by p_0, or a local maximum of

$N(p)(p-c)\theta(p)$ on the interval $[0, p_0]$. In either case, risk considerations

> *by the producer result in a lower price and higher output to the consumer, thus expanding welfare.*

The proof is elementary so we omit it. Consider an application of this model to a producer creating a new market through introduction of a new technology. Such a producers is generally a natural monopolist, yet faces great uncertainty of demand. Moreover, in this case, the potential market size N may be small, and, as a result, risk control condition may be difficult to satisfy. Our analysis indicates that utilization of risk management in their profit maximization may lead to lower prices and greater welfare. This model can provide an explanation for pricing strategies of Internet and technology startups, which, especially during the early internet shopping boom in the late 1990s offered deals that seemed too good to be true, but were true nevertheless, unfortunately also resulting in a subsequent bust of many of those firms.

Our model also provides an additional rationale for a different market design: a market created by dividing a monopolist into several smaller companies. Such "trust-busting" may expand welfare even if the resulting companies retain some degree of market power, as the resulting smaller companies will face greater uncertainty of demand.

Risk-adjusted profit analysis and welfare implications

Let us denote by $p*$ the price such that it minimizes the profit Π, not adjusted for risk (this means that the profit is calculated deterministically, taking the expected value of demand as the deterministic demand). We will now state the key assertion of this section:

Assume that Π has a unique local and global maximum at $p > c$. Then the price for which the risk – adjusted profit measure Π_R reaches maximum, is lower than the price $p*$. More specifically, for a certain range of prices, all less than $p*$, producer facing demand uncertainty enjoys higher expected risk-adjusted profit than for the previously optimal price $p*$. Moreover, increasing the price p above $p*$, provided $\Pi_R(p) \geq 0$, always (not merely in a certain range of prices near $p*$) decreases the expected risk-adjusted profit.*

We will now prove this assertion. By assumption, there is $\overline{p}$ (possibly $+\infty$) such that $\Pi_R(p) \geq 0$ for all $p \in [c, \overline{p})$. If $p* \notin [c, \overline{p})$, the assertion trivially holds. So let us consider the case $p* < \overline{p}$. Under the assumptions specified, the risk-adjusted profit is of the form

$$\Pi_R(p) = N(p)\theta(p)(p-c)\cdot\left(1 - \gamma g(p)\right).$$

Its derivative is

$$\frac{d}{dp}\Pi_R(p) = \frac{d\big(N(p)\theta(p)(p-c)\big)}{dp}\cdot\big(1-\gamma g(p)\big) - \gamma N(p)\theta(p)(p-c)g'(p).$$

As the maximum of non-risk-adjusted profit $\Pi(p)$ at $\boldsymbol{p^*}$ is unique, local and global, for every $p > p^*, \dfrac{d}{dp}\Pi_R(p) < 0$ if $g(p) \le \dfrac{1}{\gamma}$. Hence the bigger the price (as long as it is above $\boldsymbol{p^*}$ and not bigger than $\overline{\boldsymbol{p}}$), the smaller the risk-adjusted profit. Furthermore, as $\dfrac{d}{dp}\Pi_R(p^*) < 0$ and $\Pi_R(c) = 0$, there is a price $\boldsymbol{p_R}$ such that Π_R attains a global maximum at $\boldsymbol{p_R}$, with $p_R < p^*$.

The key conclusion of the above assertion is that if profit maximization is performed by considering a risk-adjusted profit measure, the result is lower prices to consumers and an increase in the Marshallian welfare measure. Furthermore, we should note that the standard comparison between a monopolistic and competitive market models gives competition as the reason that a producer must lower prices from the level of a profit-maximizing monopoly price. Our model indicates that even a pure monopolist, when faced with the risk of demand uncertainty, is likely to respond with lower prices and sacrifice of some level of previously captured profits. Furthermore, demand uncertainty results in an increase of overall welfare, due to increase in output. One could therefore venture a hypothesis that demand uncertainty is the actual force behind a transition from a monopolistic market structure to a competitive one. This hypothesis is supported by so common among monopolists desire to lock-in their markets, to assure a customer base, as the resulting reduction in demand uncertainty benefits them at the expense of the consumers.

Estimating the random demand

We will now present an efficient procedure for estimation of demand under our model. In order to estimate the function $\theta(p)$, consider a random sample $X_1, X_2, ..., X_n$ from a probability distribution defined by the condition $\Pr(X \ge p) = \theta(p)$. The random variable X is the consumer's reservation price. In order to produce such a random sample in practice, we can collect data by asking the consumers the following question:

• What is the highest price you would be willing to pay for the said good (or service)?

A simple estimator for $\theta(p)$ is then $\theta_n(p) = \dfrac{1}{n}\sum_{i=1}^{n} I_{[p,-\infty)}(X_i)$, where $I_{[p,+\infty)}$ is the indicator function. We have:

(i) $\limsup\limits_{n\to\infty}\limits_{p\geq 0}\left|\theta_n(p) - \theta(p)\right| = 0$ with probability 1, (this follows from Glivenko-Cantelli Theorem, see Fabian and Hannan, 1985).

(ii) $E\left(\theta_n(p)\right) = \theta(p)$,

(iii) $\mathrm{Var}\left(\theta_n(p)\right) = \dfrac{1}{n}\theta(p)\left(1 - \theta(p)\right)$.

It is also possible to replace the estimator θ_n with a smoothed version:

$$\hat{\theta}_n(p) = \frac{1}{n}\sum_{i=1}^{n} K\left(\frac{X_i - p}{h}\right)$$

where $K(\cdot)$ is a differentiable increasing function of a real variable (called a *kernel*), and h is a smoothing parameter depending on *n*.

Note also that maximizing $N(p)(p-c)\theta(p)$ subject to the constraint (2) (the Safety-First model case) splits into the following sub-problems:

First, estimate *N* taking into account the answers to the question A, for example:

$$\hat{N}(p) = \frac{\text{Number of consumers with } X_i \geq p}{n}. \text{ (Total Population Size)}$$

Calculate $\hat{\theta}_n(p) = \dfrac{1}{n}\sum_{i=1}^{n} K\left(\dfrac{X_i - p}{h}\right)$,

Establish the function $p \to \hat{N}(p)(p-c)\hat{\theta}_n(p)$,

Find the argument $\hat{\boldsymbol{p}}_{\boldsymbol{n}}$, which maximizes $\hat{N}(p)(p-c)\hat{\theta}_n(p)$ subject to the constraint $\hat{\theta}_n(p) \geq \dfrac{1}{1 + \hat{N}(p)\alpha^2}$.

The following convergences (almost surely, i.e., with probability 1) can be shown:

$$\lim_{n\to\infty} \hat{p}_n = p$$

$$\lim_{n\to\infty} \hat{N}(p) = N(p)$$

$$\lim_{n\to\infty} \hat{N}(p)(\hat{p}_n - c)\hat{\theta}_n(\hat{p}_n) = \max_{\theta(p)\geq \frac{1}{1-\hat{N}(p)\alpha^2}} (p-c)\theta(p)$$

under mild assumptions (see Fabian and Hannan, 1985). Similar results can be obtained for the case of risk adjusted profit measure.

Let us note that the procedure provided here by us is a relatively simple one that can be performed with relatively small quantitative power. This stands in stark contrast with existing econometric approaches (e.g., Knittel and Metaxoglou, 2014, or Blundell, Horowitz, and Parey, 2013). In the following section, we illustrate this procedure with an empirical experiment, in which we derived an estimate of a demand function for a service of online courses preparing for professional actuarial examinations.

Empirical experiment

In order to illustrate this methodology, we have performed an experiment aimed at estimating demand by students at Illinois State University for intensive review seminars preparing them for the first four actuarial examinations (professional examinations offered by the Society of Actuaries and the Casualty Actuarial Society in North America). Students were asked the question: "What would be the highest price you are willing to pay for a seminar?" separately for the first four actuarial examinations. While their responses were anonymous, they had to log in using their unique university ID, and therefore multiple entries from one person were prevented. 47 students participated. We were able to produce estimates for demands shown in the graphs below. The graphs give the probability of purchase for a given level of price, as estimated in the experiment. The students provided their responses in an online survey, with all appropriate procedures for working with human subjects secured, and with participants' anonymity assured.

The graphs are what in probability theory would be called the survival function: For every number on the horizontal axis the value given is the probability of participants being willing to pay that price or higher. This gives us the function $\Pr(X \geq p) = \theta(p)$. This, in turn, allows us to create an empirical estimate of the demand function.

Figure 1: Estimate for the seminar for the first actuarial examination
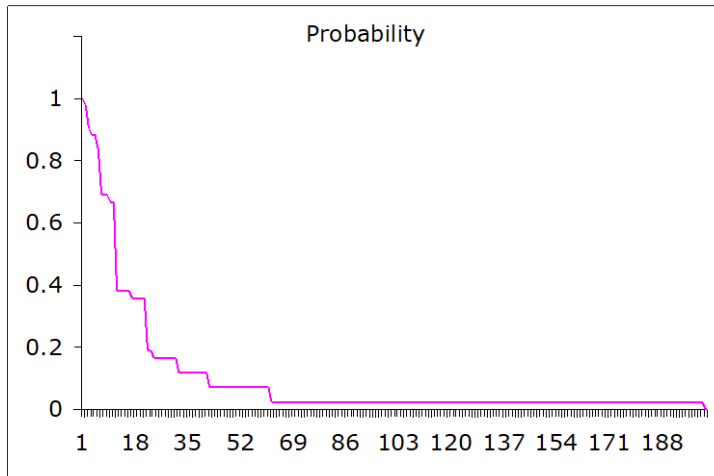

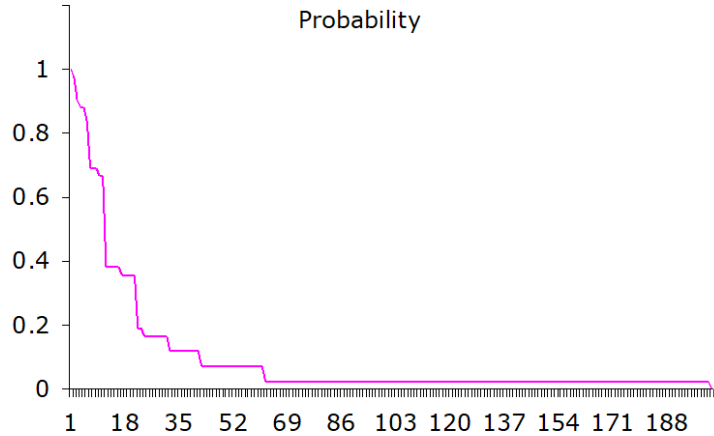Figure 2: Estimate for the seminar for the second actuarial examination

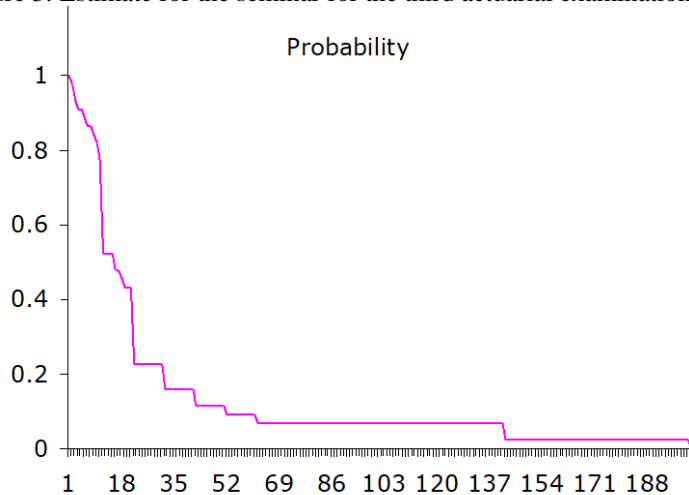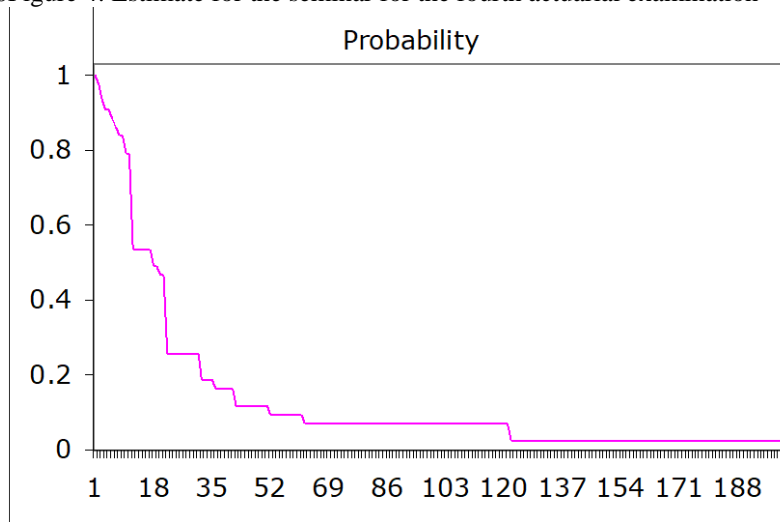Figure 3: Estimate for the seminar for the third actuarial examination

0Figure 4: Estimate for the seminar for the fourth actuarial examination

Conclusion

Our model indicates that a monopolist facing the risk of demand uncertainty will respond with lower prices and sacrifice some level of profits available in absence of such uncertainty. Furthermore, demand uncertainty results in an increase of overall welfare, due to increase in output. One could therefore venture a hypothesis that demand uncertainty is the actual force behind a transition from a monopolistic market structure to a competitive one. This hypothesis is supported by so common among monopolists desire to lock-in their markets, to assure a customer base, as the resulting reduction in demand uncertainty benefits them at the expense of the consumers. On the other hand, a monopolist facing uncertain demand effectively moves towards behaving more as a firm existing in a competitive market.

Our model provides a rational explanation for lowering price as an optimal strategy for some firms facing uncertain demand. It can be argued that such has been the behavior of major US-based airlines, which do face great amount of uncertainty of demand, and often offer mysteriously low prices and make every attempt to fly full airplanes, while European carriers, historically facing more predictable demand, used to be often content with fewer passengers. Of course that historical structure of European air travel market has been dramatically changed with the strong entrance of low-fare carriers, such as Ryanair, causing increased uncertainty of demand and lower prices. Borenstein and Rose (1994) provide an insight into the uncertainty of demand in the U.S. airline industry, and model it via price discrimination, obtaining results consistent with our model: greater price uncertainty occurs on routes with more competition or lower flight density.

Furthermore, we show that a random demand function can be derived from individual consumers' purchase decisions, through simple consumer surveys. This survey methodology offers a simple and realistic methodology for immediate application of our approach in marketing approaches of businesses dealing with uncertainty of demand.

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